The article was published in “Digital signal processing “, Moscow, No1, 2008

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THE REVIEW OF NOISEPROOF CODING METHODS WITH USE OF MULTITHRESHOLD ALGORITHMS *

The basic achievements of the noiseproof coding theory and the applied results basing at the important class of simple majority iterative methods of error correction, which have received the name the multithreshold decoders are considered. Achievement of the last ten years in the field of coding and the subsequent decoding of binary codes that is the most applicable in various high-speed satellite channels, and essentially new opportunities which have appeared at developers after detailed studying characteristics of non-binary, so-called symbolical, decoders of the same type are described. Brief comparison of characteristics of multithreshold decoders and other methods is done below: decoder Viterbi, turbo decoder and low density codes, and also some methods of error correction for Reed – Solomon’s codes which are much more difficult than majority algorithms.

Introduction

Fast growth of data processing volumes, development of digital broadcasting systems and computer networks place a serious demand on error minimization in the used discrete information. Transition of all kinds of creation, storage, use and transmission of data, and also means of an broadcasting for the digital methods, taking place now all over the world, even more raises the importance of high-quality transmission of digital streams and especially reliable storage of great data volumes. Successful work of these systems is possible only at presence of the special effective equipment which allows to ensure the proper information transmission. The major contribution to increase the reliability of digital data exchange is brought with the noiseproof coding theory. All new methods of error protection basing on use of error correcting codes are developed. The effect from their application can be in the following: they allow to increase repeatedly with other things being equal speed or distance of transmission, to reduce the sizes of very expensive aerials or to work at essentially lowered level of a useful signal in system communication. Coding application may be considered as simple as a way of multiple efficiency increase of the expensive satellite and other digital communication channels.

The present situation in decoding technique sphere is briefly characterized below. We would content ourselves with analysis of data transmission systems without a feedback from the receiver to the transmitter, channels with additive white Gaussian noise (AWGN) and linear codes which essentially facilitate realization of decoders. The basic attention we would give to the method of error correction, named multithreshold decoding (MTD). The main reasons for a deep researches choice and the analysis of multithreshold algorithms in this review are the following:

– Ability of majority methods to correct the big number of errors outside guaranteed correcting ability;

– The extremely insignificant complexity of threshold decoding procedures ;

– Property of developed MTD algorithms almost always to reach optimum decisions at rather high noise levels in a communication channel;

– Utmost ease of MTD realization even for very long codes, when just possible the achievement of maximum allowable values of coding efficiency can be.

One of the most important features of the developed multithreshold algorithms, providing their high efficiency, appears ideology of spent researches. While all efforts of modern western science representatives are directed on increase of efficiency of new error correction algorithms at less attention to questions of realization complexity, for example, expressed as decoding operations the obligatory initial requirement to new multithreshold decoding algorithms about which, basically,

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* Work is executed at financial support of the Russian Federal Property Fund (the grant №08-07-00078a)

there will be further speech, during all 35 years of their development always were extremely small number of the decoder operations both at their soft and hardware realization. So original approach to a problem of simple and effective decoding at the MTD base has given amazing results. It had been appeared, that actually always simplification of algorithms already developed by foreign experts becomes new practically unsolvable problem where decision sometimes appears really more simple than modifications of initial algorithms. However in many cases such simplifications rather reduce an efficiency of decoding in comparison with initial methods.

At the same time the principle «from simple to more effective», that always follows developers of algorithms of MTD class, has shown the huge opportunities of creation and subsequent such insignificant updating of initial methods which considerably raised their efficiency at the big noise level for many years of their research. It has also allowed to keep the minimal complexity of all decoding methods on MTD basis that rather frequently appears at equal correcting possibility essentially smaller than for competing methods, in hundreds and sometimes even in thousands times, as it will be shown further.

1. Principle of multithreshold decoding

The multithreshold decoder of self-orthogonal codes [1 – 5] is the development of the elementary Massey's threshold decoder and allows to decode very long codes with linear at code length complexity realization. In the basis of MTD work is iterative decoding that allows to come closer to the decision of the optimum decoder (OD) in enough wide range of code rates and noise levels in the channel. Thus MTD keeps simplicity and speed of the usual threshold decoder that makes it very attractive to application in existing and again created high-speed systems of communication.

Let's consider a principle of MTD work. Let us set binary linear regular block or convolutional self-orthogonal code (SOC) which is used to transmit the message with k binary symbols. After coding the total number of code symbols is equal n, $n > k$. The example of the circuit of the block SOC coder and its multithreshold decoder is submitted at fig. 1 and fig. 2. The given code is characterized by the following parameters: the code length $n=26$, the information part length of code $k=13$, code rate $R=1/2$, code distance $d=5$, a generating polynomial g (x) $=1+x+x^4+x^6$.

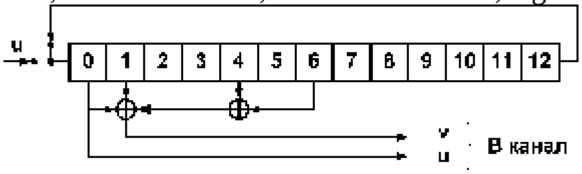

Fig. 1. The block SOC coder with $R=1/2$, $d=5$ and $n=26$

Let us necessary to transmit the information block **u**. The coding device on the basis of the information block **u** forms a code word **c = (u, v)**. As a result of transmission through binary symmetric channel (**BSC**) the decoder receives instead of a code word **c** the message disturbed by noise **y = (u', v')** with length n. First, as well as in the usual threshold decoder, the syndrome $\mathbf{s}=\mathbf{yH}^T$ (here **H** – a check matrix of a code) of the accepted message, and for each information symbol u_j, $1 \le j \le k$, they choose the set $\{S_{ju}\}$ elements of a syndrome with numbers $\{j_m\}$, named checks concerning a symbol u_j and containing as component an error e_j in this symbol.

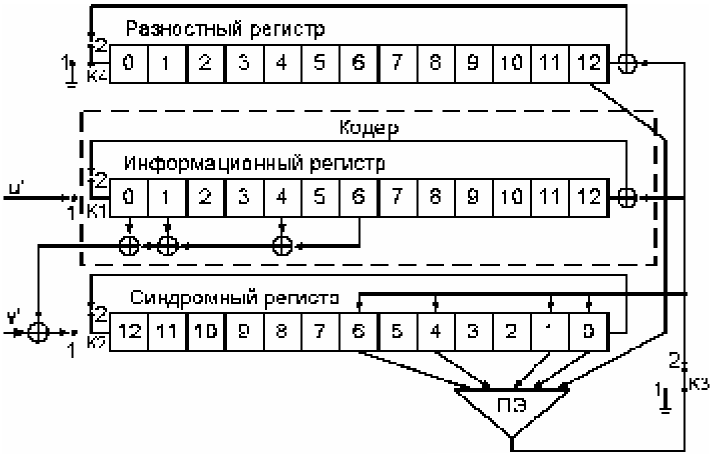

Fig. 2. The multithreshold decoder of block SOC with *R=1/2*, *d=5* and *n=26*

The binary register d with length *k*, named differencial one, originally filled by zero is introduced in addition. It is the unique distinction between the classical threshold algorithm and MTD decoder. In the register d the changed information symbols will be marked that the decoder "remembered" the message accepted from the channel and always could calculate a difference between this message and the code word which is placed in the information register.

The basic step of decoding is: for any way taken symbol u_j the function of plausibility L_j depending on checks $\{S_{ju}\}$ concerning to it and *j-th* element of the register d is calculated:

$$L_j = \sum_{\{j_m\}} s_{j_m} + d_j \qquad (1)$$

where $J=d-1$ – is quantity of checks (nonzero elements of a generating polynom g (x)); d_j – a symbol of differencial register, concerning to decoding symbol u_j (equal 0 or 1); s_p – *p-th* element of the syndrome register which is included in the set of checks relative to decoding symbol u_j.

The total number of components in (1) is equal to the minimal code distance *d*. If $L_j > T$, where $T = (d-1)/2$ is threshold value, the symbol u_j, all checks $\{S_{ju}\}$ and symbol d_j are inverted and then other symbol u_m gets out, $m \neq j$, for it the sum L_m, etc. is again calculated. If $L_j \leq T$ transition to decoding the following symbol u_m is carried out at once. Making the basic step of decoding, all *k* information symbols of the message can repeatedly get over and in any order. Thus the part of the decoder decisions on some symbols can be erroneous. Some of these errors will be corrected at the following attempts of the same symbols decoding (i.e. at the following iterations).

Basic property MTD is severe convergence of its decision to the decision optimum (on a maximum of plausibility) the decoder (OD) as at everyone *i*-th change decoding symbols the total weight of a syndrome **s** and a differential vector d necessarily decreases, i.e. there is a transition to a code word $\mathbf{c}^{(i)}$ which is more plausible, than a code word $\mathbf{c}^{(i-1)}$, which was taking place in MTD during the previous moment of the time. Passing from one word $\mathbf{c}^{(i)}$ to another, MTD can receive the most plausible word $\mathbf{c}^{(OD)}$ which is actually decision of the optimum decoder. However it is impossible to affirm, that MTD will necessarily reach the decision of the optimum decoder as in many codes admitting majority decoding, MTD on some configurations of errors weights a little bit more, than *d/2*, will stop to change information symbols earlier, than it will reach the OD decision.

One of the main reasons of it is the significant susceptibility of the threshold decoders being component MTD, to errors propagation (EP) effect [1,4]. As the result, the second and the subsequent iterations of decoding are compelled to work basically with error packets after decoders of the previous iterations, that essentially reduces efficiency of the total decoder. Hence, the basic way of MTD decision approximation to the OD decision is reduction of EP effect.

The careful choice of the codes described by a small degree of crossing of errors sets is necessary for reduction of errors duplication, included in checks concerning different information symbols, and also adjustment of parameters of the decoder (for example, values of thresholds at different iterations). The technique of an estimation of codes quality according to the mentioned above criterion is in detail described in [1, 4].

Let's note, that MTD, as well as the usual threshold decoder, is easily modified for summation of checks in (1) with some factors, in particular, at work with quantized in some levels decisions of the soft modem, which additional output bits defining reliability of the forming decision. Use of soft decisions of the demodulator allows to achieve results at 1,4 … 1,7 dB better, than at use only rigid decisions of the demodulator. Thus expression (1) for function evaluation of plausibility L_j becomes

$$L_j = \sum_{\{j_m\}} s_{j_m} w_{j_m} + d_j w_j \quad , \qquad (2)$$

where $\{w_{jm}\}$ – is the factors reflecting reliability of checks $\{S_{ju}\}$; w_j – is the factor reflecting reliability of the accepted symbol u_j.

Dependence of bit error probability $P_b(e)$ on the attitude signal / noise E_b/N_0 in the channel with AWGN at use the soft modem for the multithreshold decoder of self-orthogonal codes with code speed $R=1/2$ and various code distance d is submitted at fig. 3. The dotted line in figure shows probability of decoding error of the same codes with the help of the OD. From the submitted curves it is clear, that MTD really provides close to the optimum decoding of well chosen codes at high enough noise level in a communocation channel.

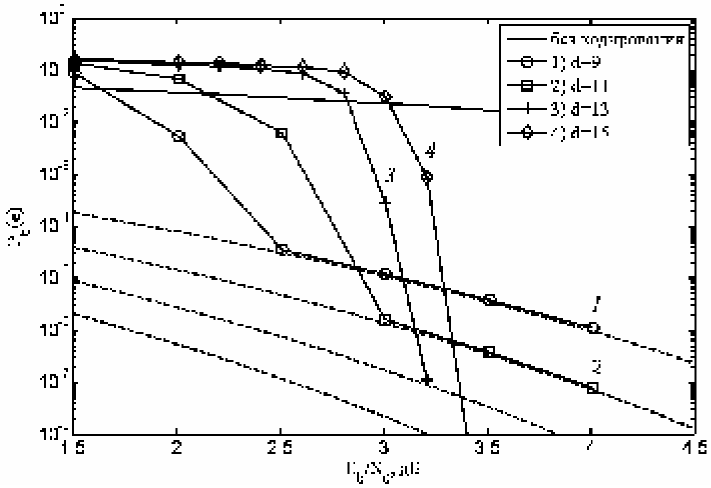

Fig. 3. Characteristics MTD for SOC with R=1/2 in the channel with AWGN

2. Complexity of multithreshold decoders realization

As it was already marked, complexity of majority algorithms can be formally appreciated as linearly growing with increase in length of the decoded code block or code restriction convolutional code.

Complexity of soft realization as number of operations per bit in binary convolutional MTD or in block SOC is equal to $N_{MTD} \approx (I+1)$ (d+2) or $N_{MTD} \approx 4d+3I$ at negligible small losses in power effectiveness (less than 0,1 dB) [5]. Here d –is the minimal code distance of a used code, I – number of decoding iterations. At decoding actually each iteration in MTD demands just a few elementary operations of addition and comparison type, and it leads to the fact, that growth of iterations number almost does not reduce real speed of the decoder, for example, in a soft variant of realization. As a good illustration of this algorithm property it is possible to refer to demoprogram MTD for convolutional code with $R=3/4$. It is used in system of special digital TV and provides for the usual PC throughout about 4 … 15 Mbit / s that is enough for processing a television color signal at rather low signal/noise ratio. Used codes pass the procedure of standardization. Demoprogram and its simple instruction can be found at at thematic web-site SRI of the Russian Academy of Sciences **www.mtdbest.iki.rssi.ru** at the educational page. Really, the comparison of MTD with other methods has shown, that speed of processing on basis MTD appears approximately at 2 decimal powers higher, than, for example, for a turbo decoders with comparable parameters of energy efficiency.

In case of hardware realization MTD, for example, with PLIS Xilinx or Altera, tests have confirmed good parameters of their efficiency at simultaneously very high throughput , up to 1,6 Gbit/sec, in particular for the decoder submitted on the cover page of this journal issue. Such opportunity has appeared after realization of the patented technical decisions for hardware MTD. According to these decisions such decoder turns in the single-cycle decision schemet, and for each step of the device clock work it can form up to 40 decisions about decoding symbols. The limiting clock frequency is determined by the greatest possible speed of the data shift accepted from the channel along decoder shift registers, with which it, basically, will consist of. Typical speed of the information move through shift registers in PLIS is limited from 100 up to 250 Mbit/s, and the number of registers working in parallel in such the MTD can exceed one hundred. It means, that productivity MTD in a hardware variant of realization can exceed essentially even 10 Gbit/sec. In essence it removes all restrictions on processing speed of such devices. These parameters of MTD efficiency provided with these algorithms makes it individual leader among all methods of high-speed digital streams transmission in expensive satellite and other channels. In particular, already developed MTD hardware versions are especially useful for systems of remote sounding the Earth as just their high-speed data streams at the limited transmitter power should be protected in every possible methods of noiseproof coding.

3. Multithreshold decoders application in circuits with parallel coding

For approximation of the limit of effective MTD work to the channel capacity, its application in circuits of parallel coding is possible [4, 5]. In the basis of construction of the given circuits allocation in SOC C_0 with code distance d_0 and code speed R_0 of certain composing code C_1 with code speed $R_1 > R_0$, also being SOC. The code distance d_1 of the allocated code is chosen by considerably smaller than d_0, and as it follows from fig. 3, the area of its effective work will be closer to Shannon's border. At parallel code decoding some iterations of decoding of the composing code C_1 are carried out, allowing approximately to decrease noticeably MTD error probability in the information sequence accepted from the channel. After that decoding process includes the rest of code C_0. Distinctive feature of the given circuit of coding is that here the external code works with code rate R_0 while in usual concatenated codes the code rate of an external code is close to one. The given property provides essential advantage MTD compare to other concatenated designs.

For an example at fig. 4 results of circuits modeling for parallel coding in the channel with AWGN for SOC with $R_0=6/12$, $d_0=13$ and $R_0=5/10$, $d_0=15$ (curves "Parallel") are submitted. In a parallel code with $d_0=13$ the external code with $R_1=6/11$, $d_1=7$ in this case has been allocated, and in a code with $d_0=15$ the code with $R_1=5/9$, $d_1=9$ has been allocated. Curves "Making" on figures are an error probability at the output of the allocated codes in the parallel circuit. Dashed lines without markers in the given figure show error probabilities of codes with $d=7$, 9, 13 and 15. For comparison at fig. 4 characteristics decoded with help MTD usual SOC with similar d and R (curves "Usual") also are shown. We must note, that characteristics for short enough codes with length up to several thousand bits, and 10 … 20 decoding iterations were used. At growth of a decoder memory size and number of fulfilled iterations with the help of parallel concatenation it has been already possible to receive the characteristics submitted at fig. of 4 curves "Long". As it follows from the analysis of the submitted curves, application of parallel coding allows MTD to work hardly more than in 1,5 дБ from channel capacity.

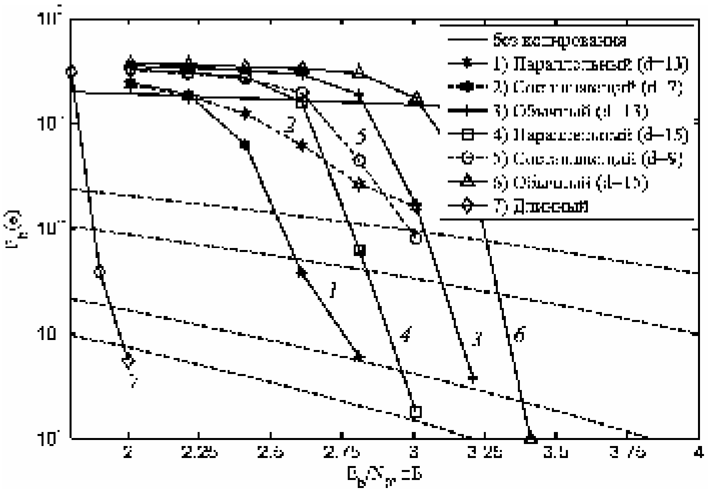

Fig. 4. Results of a parallel code modeling on MTD base in the channel with AWGN

Complexity MTD at parallel coding (in sense of quantity of carried out operations) appears even less than complexities of the usual MTD, because in this case during the first decoding iterations some elements of the syndrome register simply do not participate in sum calculations at the threshold element.

4. Concatenated codes, decoded with use of multithreshold decoders

High MTD characteristics contribute to its wide application in structure of various code constructions, because the efficiency of the last is directly linked with efficiency of their components.

The special place among code circuits on MTD base takes its concatenation with of parity check control (PCC). codes, Their usage allows to increase efficiency of coding application essentially. Feature of the given circuit consists in the fact that concatenation practically does not demand additional expenses for the equipment (for example, in the circuit of coding it is required to add only one modulo 2 adder) whereas use in a concatenation code, for example, Reed - Solomon codes is much more difficult. Principles of MTD concatenation with PCC were considered in details in [4, 5].

The effective work of the concatenation circuits consisting from SOC with $d=7$ and 9 and PCC with length 50, for the channel with AWGN is shown at fig. 5. It is apparent from the figures, that the concatenated code on MTD basis becomes much better then not concatenated. Thus use of the elementary PCC together with SOC allows to receive an additional power gain near 1 … 1,5 дБ for bit error probability at the output *of decoder* $P_b(e)=10^{-5}$. Also we shall note, that the concatenated code consisting of a code of Reed-Solomon (RS)(255, 223, 33) and convolutional code with code rate R=1/2 and length of code restriction $K=7$, decoded by optimum algorithm Viterbi, even at smaller total code rate ($R\approx 0{,}437$) concedes to the concatenation circuit on MTD base at $P_b(e) > 10^{-6}$. We shall notice, that at use of concatenation with PCC together with earlier considered parallel coding, reception of a significant additional energetic code gain is possible. The example of characteristics for one of such circuits is shown on fig. 5 by curves "MTDп+PCC". The use of the low density code in the external concatenation of the given circuit will allow to receive any small probability of an error at losses in power about 0,1 … 0,2 дБ.

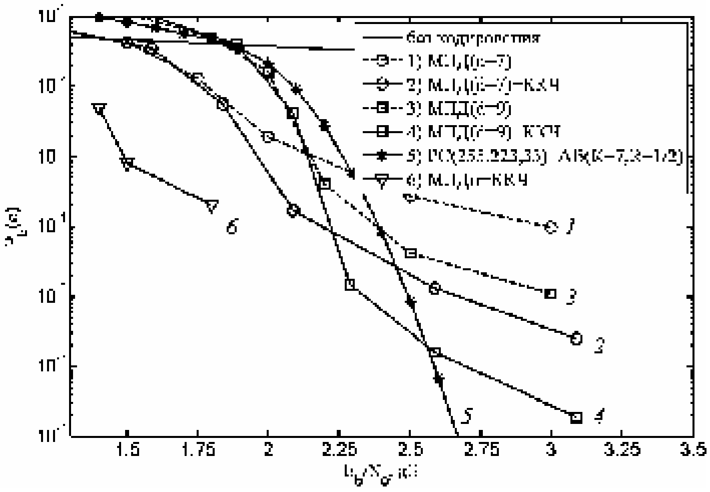

Fig. 5. Results of a code concatenation modeling for MTD base in the channel with AWGN

Complexity of decoding of the considered concatenated circuit in comparison with complexity usual MTD increases for complexity of decoder PCC, equal approximately only to two operations per information bits.

5. Multithreshold algorithms in channels with non-uniform power

Let's consider a typical example of a very simple updating of MTD algorithm, which uses its properties taking into account the concrete properties of a method very precisely and actually without complication of the method improves its characteristics of power efficiency in full conformity with a principle «from simple to effective». There is a coordination of signals system with properties of a code and the decoder.

Let's consider the two-channel circuit of transmission through satellite, Space or other channels with enough big level of Gaussian noise. We shall choose for some attitude signal/noise, originally identical to each of two chosen channels, such redistribution of the common total energy to provide maximum possible independent subsequent decoding the accepted information symbols on the basis of multithreshold decoding binary block or convolutional codes. So criterion of the best redistribution of energy between channels should be necessary a minimum level of EP display for majority decoding. In MTD theory these questions are described full enough [1, 4]. Decrease in the given effect allows to improve considerably convergence of MTD decisions to optimum ones, that creates conditions for more effective work of MTD algorithms at the big noise levels.

Creating such a simple new signal-code construction is possible to consider various ways of power balancing. For example, channels can be organized so, that through one of them information symbols of a code, and through another – check bits are transmitted. In this case the analysis of EP becomes simpler to the greatest degree, that allows to consider quickly enough and full applicability of the maximal number of codes and corresponding to them MTD algorithms in similar coding circuits. Such models have received the name of channels with non-uniform power (NUPC) [4, 11]. They can be simply realized in usual communication channels.

As the detailed analysis of some codes and some updating of MTD algorithms for NUPC channels with various parameters has shown, moving of area borders of effective MTD work aside higher noise level of the channel in a range of code rates R from 1/4 up to 3/4 can make up to 1 dB, that is very important, as already initial efficiency MTD in channels of usual type also appears rather high. Thus the attitude of channels power balance should be in a range from 1,3 up to 3,2.

Necessity of work at higher noise levels demands increase the number of decoding iterations in MTD, but such increase usually appears no more than double, that keeps small complexity of realization MTD both in program, and in hardware variants.

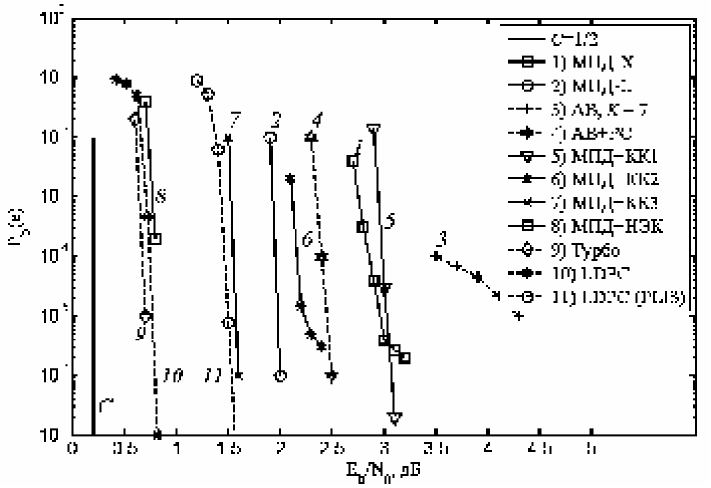

Fig. 6. Characteristics MTD, decoder Viterbi, a turbo and low density codes in Gaussian channel at R=1/2

The new received results in this area are illustrated by curves at fig. 6 on which opportunities of the suggested algorithms and already known methods are submitted. The cuve «1) MTD-X » corresponds to MTD efficiency for the decoder at PLIS Xilinx, curves «5) MTD+KK1 », «6) **MTD+КК2 » и «7) MTD+КК3** » are given for MTD application in the elementary concatenated circuits with of the of parity check control code. All of them were in details discussed in [9]. At fig. 6 curves of efficiency for algorithm Viterbi with a standard code length *K=7* (a curve «3) AB, K=7 ») and for the concatenated circuit of decoder Viterbi with Reed – Solomon code (a curve «4) AB+PC»), and also for a turbo code (a curve «9) the Turbo») [12] and low-consistence a code recommended in standard DVB-S2 [13] (a curve «10) LDPC ») also are submitted. We shall note, that at realization of the high-speed lowdencity code decoder for PLIS, where the loss in efficiency make more than 0,5 dB (a curve «11) LDPC (PLIS) »). Vertical *C*=1/2 defines capacity of the Gaussian channel that developers aspire at decoding characteristics improvement at code speed R=1/2. "MTD-L" – is a long code and MTD the decoder with *I=40* the iterations, realized in Space research institute of the Russian Academy of Sciences in PLIS Altera. The new result for MTD and channel NUPC – a dotted line «8) MTD+**НЭК** » – corresponds to an opportunity of a very simple and substantial increase of efficiency of code decoding at a delay of decision-making no more than 400000 bits , when well-known big enough speed of MTD work is kept as in soft as in hardware variant especially.

Taking into account the achieved proximity of the area of effective MTD work to a channel capacity, it is possible to count, that MTD has good prospects to reach Shannon’s border characteristics in the nearest future. This significant advantage of MTD compared to other algorithms in number of the operations, consisting of one – two decimal powers (~100 times!!!) for various combinations of coding parameters, gives the reason to believe, that it is possible to use MTD in development of the modern communications for the Space and satellite channel equipment.

6. Application of multithreshold decoders with multiitem systems of signals

In the previous sections results of MTD research in Gaussian data links with binary phase modulation (PM2) are submitted. At the same time for such channels significant restrictions at a occupied realm of frequencies are usually imposed and every year these restrictions become more and more rigid. One of the ways of an occupied realm frequencies reduction is a use of multiitem signals, to form it the peak modulation *(QAMN)* is usually applied multiitem phase *(PMN)* or quadrature. During the using such signals all approaches to MTD application remain similar to a bidimentional case that allows to receive simultaneously a significant power code gain and to save essentially a realm of a transmitted signal frequencies.

Let's consider the results of MTD modeling and other methods of error's correction in the channel with multiitem systems of signals , using soft decisions of the demodulator when the demodulator estimates reliability of decisions taken out decisions.

At fig. 7 curves «MTD, КАМ16 », « MTD, КАМ32 » and « MTD, КАМ64 » submit experimental schedules of dependence of error probability per bits P_b *(e)* at output of MTD from the attitude signal/noise E_b/N_0 in the channel with AWGN and peak quadrature modulation at use 16, 32 and 64 symbolical constellations. At decoding it was carried out from 10 up to 20 decoding iterations for block SOC with code rate R=1/2, code distance d=11 and length n ~ 10000. Thus earlier considered code with parallel coding was used.

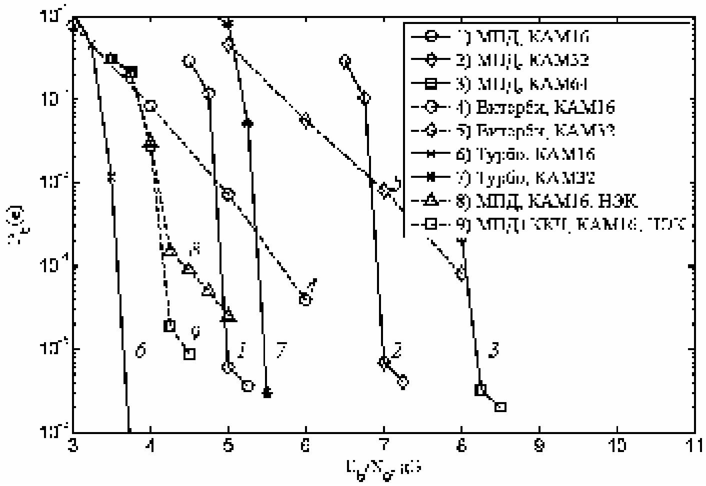

Fig. 7. MTD efficiency in channels with *QAMN*, the soft modem

In the same figure curves «AB, КАМ16 » and « AB, КАМ32 » submit characteristics of decoder Viterbi for a code with length of register *K=7* at use QAM16 and QAM32 accordingly. Apparently, decoder Viterbi in the given conditions at P_b *(e)* =10^{-4} loses MTD more than 1 dB. Curves «Турбо, КАМ16 » and « Турбо, КАМ32 » at fig. 7 are shown characteristics of very powerful turbo code [5] with code rate R=1/2 which is formed by parallel concatenation of two recursive regular convolutional codes with constructive length K=4. In the given turbo code it was applied deinterleaver such as S-random by length *L=5000* (the general length of a turbo code is *n=10000*). At decoding a turbo code 8 iterations were carried out, where for decoding codes of the each other was applied Max-Log-MAP algorithm. From comparison of the turbo code decoder characteristics a and MTD it is clear, that efficiency of the last one appears worse approximately at 1,5 dB but MTD thus almost at two decimal powers (~100 times) is easier for practical realization, than the given turbo decoder.

At the following fig. 8 characteristics MTD for the same block code, as on fig. 7, in the channel with multiitem phase modulation *(MPMN)* are submitted. They can see that parities between characteristics of the multithreshold decoder, decoder Viterbi and the turbo decoder of a code are kept for the given kind of modulation.

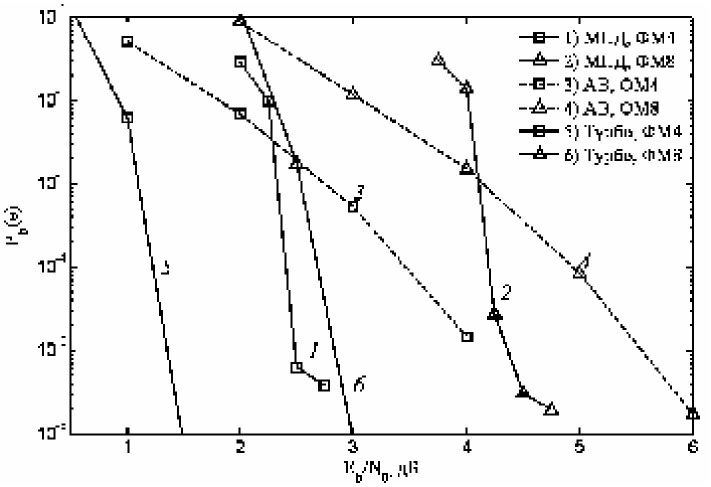

Fig. 8. MTD efficiency in channels with *PMN*, the soft modem

Let's consider one of the possible ways to approach area of effective MTD work to the channel capacity in channels with multiitem modulation. It is known, that in such channels separate bits of signal constellation are protected differently. For example, for system QAM16 at use of Grey's code of the first and third bit can be protected more, and the second and the fourth –are less. Thus the probability of an error in the first and third bits appears twice less than probabilities of an error in the second and fourth bits. The similar situation is observed and at use of other types of modulation. Thus, the channel with multiitem systems of signals can be considered as the channel with non-uniform power. In result for approach of area of effective MTD work throughput of the channel it is possible to have information symbols of the message in more reliable bits of constellation, and verifying – is less reliable.

On fig. 7 curves «MTD, КАМ16, NEK» are shown MTD characteristics block SOC with code rate R=1/2, code distance d=11 and length n ~ 10000 in the channel with AWGN at use QAM16 and the demodulator forming soft decisions, at an arrangement information bits in more reliable positions of a symbol, and check bits – are less reliable. We shall notice, that the area of effective MTD work has come closer to channel capacity approximately on 0,5 dB, but thus the area of saturation of error probability appeared a little bit above. At the same time for reduction of error probability in the field of effective work, as shown in [26], use together with MTD the elementary code with the parity check control (PCC) is possible. Characteristics of the concatenated code consisting with SOC and PCC for n=50, in case of an arrangement information bits in more reliable positions of a symbol for the same conditions are shown at fig. 7 curves «MTD+ККЧ, КАМ16, **НЭК».** From figure it is visible, that application of the offered approach has allowed to improve MTD efficiency to channel capacity approximately at 0,7 dB. In result advantage of much more complex compare to the considered turbo code (a curve Турбо, КАМ16 ») compare to MTD error probability ~ 10^{-4} appeared even smaller, than 0,5 dB. We must notice, that the similar result can be achieved and at use of other multiitem signal systems.

7. Non-binary multithreshold decoders

In many cases in real systems it is convenient to work with the data having byte structure. We shall note, that except for Reed -Solomon (RS) codes there are no others a little effective ones now with simultaneously simple decoding methods non- binary symbolical data, if the chosen code short enough.

In the given section we shall consider generalization MTD for *q-ary* symmetric channels *(qСК)*. By analogy to corresponding decoders for the binary data these algorithms are named *q-ary* multithreshold decoders *(*QМПД*)* [4, 5, 18]. They also possess property of convergence to the OD decision with preservation linear from code length of a of complexity of realization which is peculiar only to threshold procedures. QMTD value consists in that in case of great values of the basis of a code *q*, *q*> 10, it is practically impossible to create effective ODs as their complexity will be in the most cases proportional q^k where *k* – the length of information part of the code, expressed by number *q-ary* symbols.

Let's state main principles of QMTD work. Let it is set *qSC* (*q*> 2) with error probability p_0> 0 such, that by transmission any initial symbol of a code passes in one of the rest *q*-1 symbols casually, independently and equiprobably. For this channel the decision of OD will be such ones, where can be an only thing, a code word from q^k possible which differs from the message accepted from the channel in the minimal number of code symbols.

Let's consider a linear non-binary code which verifying matrix has the same kind, as well as in a binary case, i.e. will consist only of zero and units with exception that instead of 1 in an unit submatrix will be "–1". Let this matrix corresponds block SOC. As a check and generating matrix of a code contains only zeroes and ones for performance of coding operations and of the accepted message decoding it is enough to use of addition and subtraction operations on the modulo *q*. Thus, coding and decoding do not need presence of non-binary field, and it is enough to create only group of numbers, that essentially simplifies all procedures of coding and decoding. Let the decoder such as QMTD (fig. 9). It is arranged so, that after calculation by usual image of a syndrome vector **s** the accepted message decoding procedure , consisting in the idea that the next information symbol of a code controllable by non-binary threshold element u_j there is a calculation of quantity and definition of values of two concerning to it and the most frequently meeting checks of a code begins, for example, b_1 and b_2, and b_1 meets m_1 time, b_2 – m_2 time (m_1> m_2), and other values of checks for decoded symbol u_j meet no more m_2 time. Then at the output of *q-ary* threshold element (**QПЭ***)* there will be a value b_1 and QMПД at each change of a symbol u_j will pass to more and more plausible decisions as thus the number of distinctions between the code word corresponding to the current contents of the information register, and the sequence accepted from the channel will decrease. If itvappears, that two the most frequently checks meeting value are those, that m_1=m_2 the threshold element (TE*)* output is established equal to zero, i.e. the symbol u_j does not change, and attempt of decoding of any other information symbol of a code is done. The most essential circumstance raising adjusting properties described QMTD, the opportunity to make the correct decisions is at great values *q* all at two correct checks be relative u_j from *d*-1 possible, that occurs in that case when all wrong checks of the decoded symbol u_j have various values.

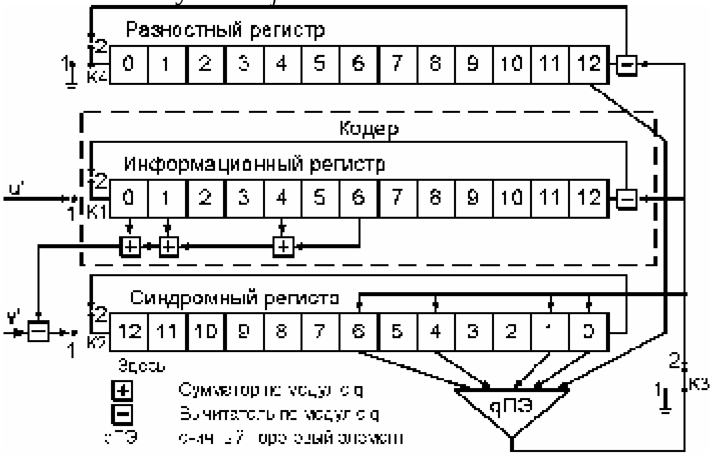

Fig. 9. QMTD of the block SOc with R=1/2, d=5 and *n=26*. The top inputs of cells 0, 1, 4 and 6 registers of a syndrome are subtracted on the modulo *q* from contents of corresponding cells

Further results of QMTD work modeling in non-binary symmetric channel *q*SC, characteristics of code concatenations for QMTD with codes of the control on mod *q*, and also opportunities of usual decoders of codes PC are considered. We shall note, that as against Rid-Solomon codes for *qMTD* any restrictions on length of a code in general are not present, as the length of a code and size of its basis in non-binary codes with majority decoding do not depend at all from each other. The volume of modeling in the bottom points of curves for QMTD made from $5{\cdot}10^{10}$ up to $2{\cdot}10^{12}$ bits, that testifies to extreme simplicity of a method.

At fig. 10 dependences of error probability per symbol $P_s(e)$ and probability of symbolical error P_0 in qSC for RS codes which are designated as RSn where n – the length of a code expressed among symbols are submitted. We shall note, that only RS codes of length 255 or less really now are used. Codes with length n= 4095 symbols and, especially, with lengths n=*65535* (each symbol – the size 16 bits), in the foreseeable future are not subject to realization.

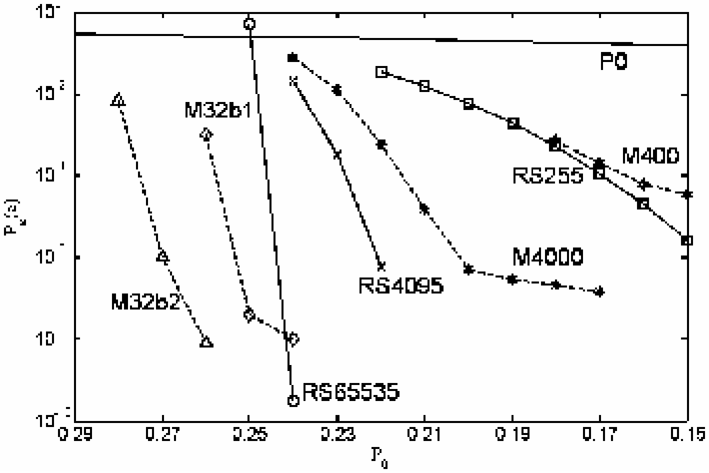

Fig. 10. Characteristics of the non-binary block MTD and codes RS with *R=1/2*

Here dashed lines show opportunities of codes with majority decoding at R=1/*2* for a case *q=256* (a symbol – one byte) for different code lengths at anyone *q*> 2, as well as in a binary case, for QMTD it is possible to build as much as long codes with various values of code distance ***d*** and code rate R. These codes are marked as M400 and M4000 with the numbers designating code lengths, expressed by number of symbols. Further, designation M32b1 corresponds QMTD for a code of length of 32000 symbols. Apparently from fig. 10, opportunities of QMTD in all cases are comparable or better, than at rather complex decoders of RS codes. Moreover, very simple for realization the last decoder for a code length n=32000 appears capable to provide with the elementary majority methods a noise stability essentially unattainable even for code PC of length of 65535 two-byte symbols, which the decoder will not be created never for. And if to proceed two-byte non-binary codes with majority decoding its characteristics for code length n= 32000 symbols will correspond to curve M32b2, to even more indicative on a level of a noise stability in area where RS codes any more do not work. Thus QMTD for two-byte symbols actually in anything is not more complex than one-byte as even usual microprocessors equally easy and quickly work and with one-byte symbols, and with 2 and even sometimes with 8-byte words.

AT fig. 11 opportunities QMTD and RS codes in *qSC* are shown at code rate R=7/8. Continuous lines with the indication of block lengths submit error probabilities per a symbol for RS codes. Dashed lines submit codes with *qMTD* decoding and in the length of 48000 symbols: b1 – byte (a symbol – 8 bits) and b2 – two-byte (a symbol – 16 bits). We must emphasize, that the opportunity of RS code decoder creation for length n=4095 at R=7/8 in the nearest future remain very problematic while even for codes of length n=48000 bytes considered non-binary majority decoders remain very simple.

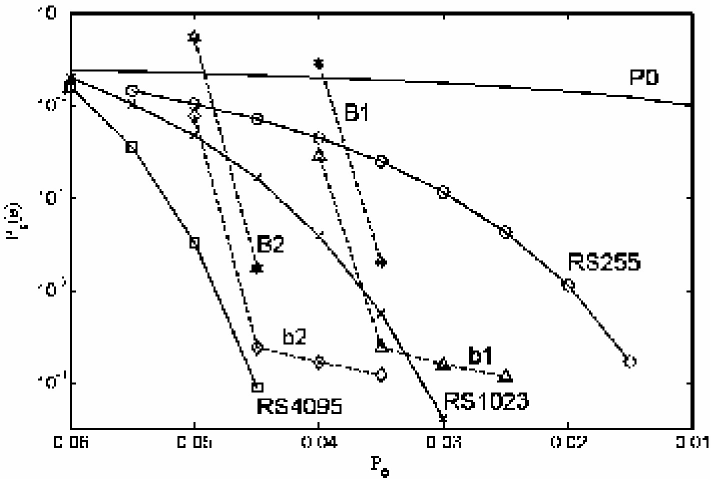

Fig. 11. Characteristics of RS codes and QMTD with R=7/8

AT fig. 12 for codes with small redundancy at R=0,95 similar characteristics for QMTD and RS codes are submitted. For comparison At fig. 12 the curve for RS code with n=255 and R=7/8 from fig. 11 is placed also. Dotted lines b1 and b2 specify opportunities of two QMTD for codes with length n~80000 and symbols in the size 1 byte and 2 bytes.

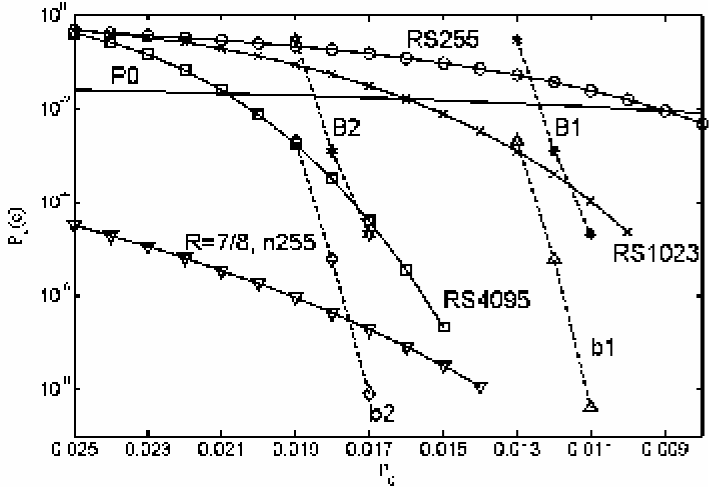

Fig. 12. Characteristics QMTD and RS codes at R=19/20

From comparison of RS codes with length n=255 at R=7/8 and R=19/20 they can see, as far as the last one is less effective then the first and as far as it is more difficult to provide good efficiency at the redundancy reduction. Nevertheless characteristics of low density codes with majority decoding on the QMTD basis appear rather high and can lift essentially a level of a noise resistance if the chosen codes have big enough lengths. Moreover, QMTD at R=0,95 as follows from fig. 12, is more effective than RS code with R=7/8, where redundancy in 2,25 times more. It removes all questions on application of any new the most complex methods of decoding for RS codes: they are ineffective in comparison with QMTD. In particular, for example, algorithms of Sudan and other serious complications of decoders for RS codes result in their growth asymptotic complexity with code length from a level n^2 up to size about n^3. Thus in the best case for these algorithms the weight of error of the channel at which reception of the correct decision is possible, grows at use of the most complex algorithm of this class less than at 4 % for R=7/8 and less than at 1 % for R=0,95, i.e. for those values of rate which are submitted at figures. It is necessary to repeat, that fig. 12 shows the superiority QMTD with ***R=0,95*** before RS code of length 255 even with R=7/8, i.e. at its guaranteed redundancy at 125% greater, than at R=0,95. Therefore growth of Sudan algorithm redundancy at 4 % and 1 % even for long RS codes will give nothing comparable on length with codes, chosen for QMTD.

It is useful to note, that everywhere for curves b1 and b2 the bottom experimental points correspond OD (with total searching!) with single error at the frequency of their occurrence close to estimations, submitted in [4 – 6]. Other experimental data for QMTD can be found in [4 – 6, 17].

8. Characteristics of concatenation of non-binary multithreshold decoders

According to the general principles of the coding theory, transition to concatenation principles of coding will improve characteristics QMTD even more. Thus complexity of decoding will increase in comparison with initial algorithm very insignificantly.

Opportunities of concatenation QMTD with use codes of the control on mod q control also are submitted for high-speed codes an fig. 11 and fig. 12. Errors probabilities per the block for concatenated codes with internal SOC code at R=7/8 and an external code in length L=190 are submitted to curve B1 for a code with $q=256$ (a symbol – 1 byte) and curve B2 for a code with $q=65536$ (a symbol – 2 bytes) at fig. 11.

For a concatenated code with an internal code at R=0,95 and an external code with L=190 the error probability per the block at fig. 12 is given by curves B1 for q=256 and B2 for $q=65536=2^{16}$. In all cases on curves B1 and B2 the bottom points correspond to size N_B^{-1}, where N_B – is number of the decoded blocks as in experiments there was no case of wrong decoding of a concatenated code in these points. Predictably, application of concatenation at many decimal powers reduces error probability per block in comparison with usual QMTD almost without some appreciable growth of redundancy of a concatenated code. Thus the increase in volume of calculations in a concatenated code makes less than 20 % in comparison with initial algorithm QMTD.

The further significant decoding efficiency improvement by QMTD methods is possible at transition to convolutional codes, methods of consecutive and parallel concatenation coding, application of codes with the allocated branches and to other measures, some part of which are described in [3 – 5].

9. Complexity of realization of non-binary multithreshold decoders

Consideration non-binary (symbolical) QMTD shows, that linear decoding complexity is kept. At soft realization of such algorithm the subroutine of its threshold element work which represent practically total decoder, occupies less than ten short text lines in C++ and provides processing simultaneously such number of bytes in accepted or kept messages which are supported by architecture of the used processor. It increases even more and so the highest usually throughput of MTD class decoders. Demoprogram for symbolic QMTD is also laid out for common use on a web–site www.mtdbest.iki.rssi.ru . It shows at the smallest redundancy (R=0,95) and for very large error symbol probabilities in non-binary channel (more than 10^{-2}) practically optimum decoding of very long codes with a speed more than 10 Mbit /s, and sometimes at the rather fast general purpose PC – up to 30 Mbit/s. Thus demoprogram performs full work imitation of for data transmission: formation an information stream, its coding with the chosen formats, entering the significant noise distortions in messages and then work of discussed decoding algorithm. So real soft QMTD decoder throughput the can be considered as the greater one in ~ two – three times or even more. Demoprogram of the classical RS code decoder is also placed at the web-site www.mtdbest.iki.rssi.ru .

Complexity of decoders for long non-binary codes is natural comparing to computing expenses of decoders of RS codes which grow as n^2. More simple decoders for non-binary codes, except for MTD, are unknown. Various methods of increase of RS codes efficiency, including all variations of algorithm of Sudan, result in complexity about n^3. For codes with length n~30000 symbols it results in a complexity difference with QMTD ~ $n^2=30000^2\approx10^9$, i.e. billion times. However an improvement of a noise resistance due to the more complex decoding, for example, at R=7/8 is rather insignificant. It is defined by the fact, that the improvement corresponds as though to increase in the minimal code distance of RS codes approximately in 1,04 times. Certainly, it does not happen really, but the cases when it is not so, and the decoder is mistaken at smaller weight of error, is insignificant. For this case estimations show, that RS code with length n~4095 symbols at R=7/8 will provide error probability per a symbol ~10^{-6} at error probability in the channel about p_0~0,050, the Sudan algorithm will be so effective at $p_0 \approx 0,052$ whereas QMTD provides the same probability per the block at $p_0 \approx 0,047$ for a code with length n~10^5 symbols. At use with symbolical MTD with PCC, MTD characteristics will be better than concatenation almost without decoder complexity growth. Thus, characteristics of all three methods are close, but even usual RS decoder will be in a few thousands times , probably, in 10^4 times more complicated on operations number than QMTD. The Sudan algorithm in case of its realization for the same RS codes, will demand still approximately at three decimal powers greater number of operations, than usual RS decoder, that in general removes from discussion questions of its practical use. Let's note, that for R=1/2 and R=0,95 the resulted curves show absolute advantage of non-binary MTD in comparison with RS codes of any reasonable length. It is testified also by demoprogram of different methods of non-binary codes decoding, described on the website web-site www.mtdbest.iki.rssi.ru .

Conclusions

MTD application in satellite and other expensive channels allows to realize any high speeds of processing and to increase essentially its performance efficiency. The extremely simple MTD device in comparison with other comparable with it on efficiency methods makes their preferable in fast broadband channels at hardware realization. In enough slow channels even soft MTD realizations are very effective and demand a spelling only of a several tens commands of a program code for a threshold element. Simple methods of the code and signals coordination even more raise MTD opportunities and make their realization especially simple. Completely insignificant difference in MTD efficiency in comparison with some especially complex decoders of other types, as dynamics of improvement of MTD characteristics shows the last years, will be overcome, probably, in the near future.

Except the natural areas of simple highly effective methods of coding in communication networks, it is necessary to note good opportunities of QMTD application_for coding the information in CD,DVD and other carriers of great information volumes, in the super big bases of audio- and video- data with much higher level of reliability, than it was accessible until recently, and also at updating, restoration, carry and use of the data kept there. Thus it is easy to provide the operative constant control over quality of the kept information, and also duly updating and carry of the data owing to ageing and arising defects of the carrier. All kinds of the dynamic control of a reliability level, memory management and its reservation by the most obvious way may be realized on the fact basis that_QMTD_algorithms continuously carrying out various simple but very informative and convenient majority estimations of reliability of the written down data. It also defines all side benefits of multithreshold algorithms in their appendices on maintenance essentially new, on many decimal orders of higher level of integrity and reliability of storage of the information in the super big data files with practically arbitrary structure.

Thus, essentially new level of a noise resistance achievable with the help different types MTD algorithms allows to solve problems of maintenance of high reliability of data transmission and storage without any additional completion of these algorithms or only at their insignificant adaptation to the possible additional requirements arising in large-scaled digital systems.

Development of MTD algorithms were supported by Scientific council on a complex problem "Cybernetics" AS USSR, scientific research institute " KVANT" and SRI of the Russian Academy of Sciences. The additional information about different classes of MTD is located on specialized thematic bilingual website SRI RAS www.mtdbest.iki.rssi.ru .

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