THE SUMMARY

This monograph was issued in 40-year jubilee of publishing in Russian surprising on the depth and simultaneously on statement's clearness J. Massey's book «Threshold Decoding», 1966. Believing this concurrence important and obliging, the author has increased his efforts in preparation for the publication his researches about further development of extremely simple linear codes majority decoding methods. Therefore the author hopes, that at least some of those multithreshold decoding (MTD) methods which have been submitted in the given book, appear really useful for communications of a new century.

We list briefly those main new results which propose iterative MTD methods on leading positions in global competition between error correcting algorithms for channels with a large noise level.

First, the book about MTD represents the investigation where the unity of block and convolutional codes is constantly underlined especially at the application level.

Undoubtedly, it is the proof of key MTD algorithms properties according to which decoding symbols changes always lead to strictly more probable decisions. Any analogues of such significant properties for other error correcting algorithms are not known until now.

Found classes of codes are almost not subjected to effect of error propagation (EP), i.e. grouping of errors at the output of the threshold decoder. All used earlier approaches to studies of EP effect could not give anything constructive for the idea of repeated error correction.

Let's emphasize, that in many cases MTD decoder at rather high noise levels reaches the optimum decoder decision. At the same time, though achievement of optimum decoder decisions usually demands total search methods, complexity of algorithm MTD grows with length of a code just linearly.

Nevertheless, MTD is not an optimum method and, after the description of the two most complicated problems decision: search for a simple good algorithms and choice of the most suitable for them codes, – the most part of the book is really devoted to methods of overall MTD performance increase at possibly higher noise level. Thus all methods of algorithms improvement offered for such MTD are always strictly selected by criteria of the minimal complexity as at their estimation from the point of view of operations number, and at calculations of hardware realization throughput.

For the various codes used in MTD, and the majority of decoding variants on the basis of this method useful estimations of efficiency are resulted. In the case of need in additional estimations of efficiency of codes and algorithm MTD it is possible to address to our reference book about codes, at specialized website SRI of the Russian Academy of Sciences **www.mtdbest.iki.rssi.ru** or to imitate that circuit which causes questions, on a computer.

Moreover, on this website it is possible to find the detailed answers on questions asked frequently by the readers, wishing to improve their understanding of error correcting coding problems.

Estimations of complexity of program realization show MTD advantage compare to other methods at ~2 decimal exponents in operations number per bit at comparable efficiency. It is a very rare case in a history of digital processing methods development. They should be used in a proper way. In the channel with rather large noise level at modeling MTD work with usual personal computer its throughput is more than 1 Mbit/s per 1 GHz the processor clock frequency that exceeds extremely throughput of other soft algorithms at the same signal/noise efficiency.

Now codes are constructed and soft MTD processors are created and accepted to the standardization for digital channels of special television system. It is difficult to remember other similar decisions by domestic development of digital communication techniques. For the specialized microprocessors speed of MTD decoding can be increased additionally.

Therefore it is improbable, that any other effective enough methods can be simplified in the same way.

Results of hardware realization MTD on a basis PLIS Xilinx and Altera show an opportunity of achievement an absolute degree of parallel calculations. In this case the chip architecture can be built in such a manner that the decoder seems does not spend for operations with the data of the syndrome register, generally, any time at all. Directly at the moment of shift data cycle end through registers of the decoder the decision on error value in decoding symbols always are already done. Therefore throughput of MTD algorithm can be determined only by the greatest possible speed of simultaneous data shift in all such memory registers of which ones almost completely this decoder will consist. This opportunity is already successfully realized on standard PLIS due to creation of corresponding adaptive majority element circuit and selection necessary differentials in generating code polynomials. Such two stage parallelism in MTD operations forms its super fast work. Thus, advantage of hard MTD compare to other algorithms at speed at 2÷3 decimal exponents is already achieved also.

MTD simply decode very long codes for which ones only effective realization of error correction is possible at the large noise of the channel.

The codes and their very simple MTD decoders without operations of multiplication and division for non-binary channels also were offered 20 years ago. At relatively close on noise level conditions MTD is at 2…3 decimal exponents and better than characteristic of error probability for Read-Solomon (RS) code decoders. It is connected, as well as in binary codes, by that MTD successfully, very simply and almost in optimal manner decodes rather long codes, which ones cannot be constructed principally in a class of RS codes. We shall notice that among non-binary codes it is very difficult to create effective Vertebra algorithm also. So non-binary MTD is a fine example of completely unique and extremely effective decoder for which any other non-binary codes and the good decoders of other class really does not exist at all. And as the huge number of various variants of codes PC application now is known it also means, that the need for non-binary codes is extremely great, and all opportunities are limited by codes RS possibilities. For example, non-binary MTD provides the simplest way to increase data reliability stored on CD-ROMs or the transmitted digital data at many decimal exponents. These decoders form essentially new level of quality and integrity of digital symbolical data streams that are unattainable for RS decoders at all. And the extremely important, non-binary MTD is very simple also, as well as its binary analogues.

MTD is possible to apply and for simultaneous with error correcting coding data compression, in particular, with binomial source statistics. It is very important, that for some types of sources compression with MTD is realized at a level very close to theoretically limiting possibilities. And, the very essential, such MTD are not afraid even of high error density in the input packed streams. In this case it restores the data with required high quality also. There are not finding out any attributes of «fragility» of the compressed information at all when distortions in the transmitted data lead to the big packets of error in the restored unpacked information.

In channels with erasures MTD work almost at the capacity of such a channel, at many decimal exponents reducing a remained number of the erased symbols in comparison with their initial density in input digital stream. It seems to be almost unattainable for other methods also. And, restoring erased data for MTD is even easier task, than for decoder in binary symmetric channel, though complexity of MTD for errors is very insignificant too.

Concatenated circuits for MTD are unique. At concatenating with parity check codes sometimes it is possible to get with one –two additional decoding operations saving all of the advantages on concatenation efficiency. It is much more simple in complexity than decoding of concatenated circuits with codes RS which ones sometimes are less effective also.

Further, almost always it is possible to make so that at the second decoding stage of a concatenated code with MTD code rate of this error correction stage actually coincided with speed R_0 of parallel concatenated code as a whole. Usually the second-order decoder in the consecutive concatenated circuit works at speed $R_1 \approx 0{,}8\ldots0{,}95$, i.e. it is initially much less effective than a code with code speed of full code R_0. Substantially for this reason MTD for concatenated codes are especially effective, remaining thus almost so simple, as well as usual main MTD algorithms. These effective parallel MTD coding schemes probably have appeared much earlier than all other similar parallel concatenating methods.

MTD works well with various compound multipositional signal systems which ones considerably compress a spectrum of a transmitted signal with respect to traditional binary keying FM2.

Useful results are received for MTD at use of codes with unequal bits protection, with non-uniform power of channels, in case of MTD application for codes with allocated branches. So MTD decoders easily adapt to various conditions of their application in communication systems.

Let's notice, that at absence of the soft modem which ones are necessary for the majority of other effective algorithms, MTD in this case will be very essentially simplified even more, and some decrease in its characteristics at signal/noise level will be very moderate.

The main step of designing MTD – optimization of its many hundreds of parameters. It is a good example of powerful modern computers and technical equipment correct use and the same methods of adaptation and optimization which realizes MTD itself, in designing these extremely useful algorithms. Thus additional increase of resulting reliability of decoding achieves sometimes 1÷2 decimal exponents without any increase in operations number in a final variant of such decoder after performance optimization is made. Any other methods of error correction have no such really powerful additional means of efficiency increase at all.

In general total MTD ideology is based on ideas of optimization functional of very large number of variables at all three design stages. First it needs at a choice of codes for MTD, at the stage of decoder optimization and at real work in noisy channel. Note that optimization of weight values of checks, thresholds and separate differential parameters of code polynomials in the chosen decoders requires large efforts, but gives most important results. This complex optimizing means usage provides high decoder performance.

Opportunities MTD on the coordinated mutual exchange between values of its parameters are very wide: memories, decision delays, number of operations, throughput, code length, redundancy, noise level of the channel and a code gain. Always it is possible to choose such parameters of the decoder, that under any practically consistent technical requirements for its development it is possible to create good MTD device. For example, for achievement of the maximum speed it is possible to realize high-efficiency convolutional MTD due to increase memory and delay of the decision in decoder. If it is necessary to decrease in a delay of the decision then they should apply block codes that reduces throughput of the decoder, and at small transmission speed choice it is possible to load successfully the decoder with more powerful algorithm of correction with more comprehensive threshold element which will allow to work at the greater noise level, etc.

At last, we shall emphasize, that all these fundamental results are consequence of serious theoretical researches, developments within many decades of very powerful tool, test benches and the software for researches, tests and the control of work MTD in various PLIS, and also for creation digital channels simulators and models of new decoding algorithms.

In connection with the technology of designing MTD decoders described above there is one more major aspect of multithreshold decoding researches. The matter is that together with 35-years researches on subjects MTD all these years hardware-software

means were created which changed the base computers, the purposes and reference points, quickly changed according to fast development of digital technical equipment. Now specially created system programs for researches, optimization and designing MTD makes effective software that continues to develop intensively. Under such conditions full and all-round deep expert education in the field of researches and development of algorithms MTD which is carried out on many directions allows to continue successful researches of multithreshold decoding on subjects.

The part of the questions considered in the book is analyzed in our reference book on coding «Error correcting coding. Methods and algorithms», Moscow, publishing house «Hot Line –Telecom», 2004. Internet-shops where it is possible to buy it, easy to find through search system **www.findbook.ru** or to get it directly in publishing house **www.techbook.ru**, phone number +7 495 737 39 27 in Moscow.

The latest information on MTD may be found at specialized web-site Space Research Institute of Russian Academy of Sciences on codes **www.mtdbest.iki.rssi.ru**. It is updated regularly.